

Symmetry breaking in light-front ϕ^4 theory^a

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Abstract

We consider the symmetric and broken phases of light-front ϕ^4 theory in two dimensions. In both cases the mass of the lowest state is computed and its dependence on the coupling used to infer critical coupling values. The structure of the eigenstate is examined to determine whether it shows the signs of critical behavior, specifically whether the one-body sector becomes improbable relative to the higher Fock sectors. In attempts to establish this behavior, we consider both sector-independent and sector-dependent constituent masses.

^a Based on a talk contributed to the Lightcone 2016 workshop, Lisbon, Portugal, September 5-8, 2016.

I. INTRODUCTION

We apply a new computational method to light-front [1–4] ϕ^4 theory in two dimensions, in both the symmetric and broken phases [5–10]. The method is based on an expansion of the Fock-state wave functions in a basis of multivariate symmetric polynomials [11, 12]. This allows fine tuning of the resolution, Fock sector by Fock sector, and incorporates small- x behavior that captures an integrable singularity. Both features represent an improvement over the traditional discrete light-cone quantization (DLCQ) [13, 14] approach, where the resolution is fixed across all Fock sectors and the integrable singularity at zero momentum fraction is ignored. The presentation here extends earlier work in [15].

The general method and an application to the broken phase are described here; the symmetric phase is discussed specifically by Chabysheva [16]. Section II details the structure of the ϕ^4 eigenvalue problem and our method of solution, including the option of a sector-dependent mass. Results are presented in Sec. III, followed by a brief summary in Sec. IV

II. EIGENVALUE PROBLEM

The Lagrangian for ϕ^4 theory is $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\mu_0^2\phi^2 - \frac{\lambda}{4!}\phi^4$. From it, one obtains the light-front Hamiltonian density $\mathcal{H}^- = \pm\frac{1}{2}\mu^2 : \phi^2 : + \frac{\lambda}{4!} : \phi^4 :$ and the Hamiltonian $\mathcal{P}^- = \int dx^- \mathcal{H}^-$ that defines the eigenvalue problem $\mathcal{P}^-|\psi\rangle = \frac{M^2}{P^+}|\psi\rangle$. Here, $P^+ = E + p_z$ is the light-front momentum conjugate to the light-front spatial coordinate $x^- \equiv t - z$, and \mathcal{P}^- is the operator that generates translations in light-front time $x^+ \equiv t + z$. The mass of the eigenstate $|\psi\rangle$ is M .

The Hamiltonian is computed with the mode expansion

$$\phi(x^+, x^-) = \int \frac{dp^+}{\sqrt{4\pi p^+}} \left\{ a(p^+)e^{-ipx} + a^\dagger(p^+)e^{ipx} \right\}. \quad (2.1)$$

The creation operators $a^\dagger(p^+)$ satisfy the commutation relation $[a(p^+), a^\dagger(p'^+)] = \delta(p^+ - p'^+)$.

However, before completing the construction of \mathcal{P}^- , we consider an asymmetric form obtained by a shift in the field. The generator for the shift is $U = \exp \int dp^+ [f(p^+)a^\dagger(p^+) - f^*(p^+)a(p^+)]$. This shifts the creation operator $Ua^\dagger(p^+)U^\dagger = a^\dagger(p^+) + f^*(p^+)$. Following Harindranath and Vary [17], we choose f to correspond to a zero mode $f(p^+) = f^*(p^+) \equiv \sqrt{\pi p^+} \delta(p^+) \phi_s$. The field is then shifted by a constant, $U\phi U^\dagger = \phi + \phi_s$, and, with $\phi_s = \pm\sqrt{6\mu^2/\lambda}$, we obtain

$$U : \mathcal{H}^- : U^\dagger = -\frac{3\mu^4}{2\lambda} + \frac{1}{2}(2\mu^2) : \phi^2 : + \frac{\lambda\phi_s}{3!} : \phi^3 : + \frac{\lambda}{4!} : \phi^4 : \quad (2.2)$$

Now substitution of the mode expansion yields $\mathcal{P}^- = \mathcal{P}_{11}^- + \mathcal{P}_{22}^- + \mathcal{P}_{13}^- + \mathcal{P}_{31}^- + \mathcal{P}_{12}^- + \mathcal{P}_{21}^-$, with

$$\mathcal{P}_{11}^- = \int dp \frac{2\mu^2}{p} a^\dagger(p)a(p), \quad (2.3)$$

$$\mathcal{P}_{22}^- = \frac{\lambda}{4} \int \frac{dp_1 dp_2}{4\pi\sqrt{p_1 p_2}} \int \frac{dp'_1 dp'_2}{\sqrt{p'_1 p'_2}} \delta(p_1 + p_2 - p'_1 - p'_2) a^\dagger(p_1) a^\dagger(p_2) a(p'_1) a(p'_2), \quad (2.4)$$

$$\mathcal{P}_{13}^- = \frac{\lambda}{6} \int \frac{dp_1 dp_2 dp_3}{4\pi \sqrt{p_1 p_2 p_3 (p_1 + p_2 + p_3)}} a^\dagger(p_1 + p_2 + p_3) a(p_1) a(p_2) a(p_3), \quad (2.5)$$

$$\mathcal{P}_{12}^- = \mu \sqrt{\frac{3\lambda}{2}} \int \frac{dp_1^+ dp_2^+}{\sqrt{4\pi p_1^+ p_2^+ (p_1^+ + p_2^+)}} a^\dagger(p_1^+ + p_2^+) a(p_1^+) a(p_2^+), \quad (2.6)$$

$$\mathcal{P}_{31}^- = (\mathcal{P}_{13}^-)^\dagger, \quad \mathcal{P}_{21}^- = (\mathcal{P}_{12}^-)^\dagger. \quad (2.7)$$

The eigenstate of \mathcal{P}^- , with eigenvalue M^2/P^+ , can be expressed as an expansion

$$|\psi(P^+)\rangle = \sum_m P^{+\frac{m-1}{2}} \int \prod_i^m dy_i \delta(1 - \sum_i^m y_i) \psi_m(y_i) |y_i P^+; P^+, m\rangle \quad (2.8)$$

in terms of Fock states $|y_i P^+; P^+, m\rangle = \frac{1}{\sqrt{m!}} \prod_{i=1}^m a^\dagger(y_i P^+) |0\rangle$ with normalization $1 = \sum_m \int \prod_i^m dy_i \delta(1 - \sum_i^m y_i) |\psi_m(y_i)|^2$. The eigenvalue problem is then reduced to a coupled system of equations

$$\begin{aligned} & \left\{ \begin{array}{c} +\mu^2 \\ -\mu^2 \\ 2\mu^2 \end{array} \right\} \sum_i^m \frac{1}{y_i} \psi_m(y_i) + \frac{\lambda}{4\pi} \frac{m(m-1)}{4\sqrt{y_1 y_2}} \int \frac{dx_1 \psi_m(x_1, y_1 + y_2 - x_1, y_3, \dots, y_m)}{\sqrt{x_1(y_1 + y_2 - x_1)}} \\ & + \frac{\lambda}{4\pi} \frac{m\sqrt{(m+2)(m+1)}}{6} \int \frac{dx_1 dx_2 \psi_{m+2}(x_1, x_2, y_1 - x_1 - x_2, y_2, \dots, y_m)}{\sqrt{y_1 x_1 x_2 (y_1 - x_1 - x_2)}} \\ & + \frac{\lambda}{4\pi} \frac{(m-2)\sqrt{m(m-1)}}{6} \frac{\psi_{m-2}(y_1 + y_2 + y_3, y_4, \dots, y_m)}{\sqrt{y_1 y_2 y_3 (y_1 + y_2 + y_3)}} \\ & + \mu \sqrt{\frac{3\lambda}{8\pi}} m \sqrt{m+1} \int \frac{dx_1 \psi_{m+1}(x_1, y_1 - x_1, y_2, \dots, y_m)}{\sqrt{x_1(y_1 - x_1) y_1}} \\ & + \mu \sqrt{\frac{3\lambda}{8\pi}} (m-1) \sqrt{m} \frac{\psi_{m-1}(y_1 + y_2, y_3, \dots, y_m)}{\sqrt{y_1 y_2 (y_1 + y_2)}} = M^2 \psi_m(y_i), \end{aligned} \quad (2.9)$$

where the last two terms are kept only for bottom option of $2\mu^2$, which represents the shifted, asymmetric form of the Hamiltonian. In the symmetric phase, we consider only the odd eigenstate where the number of constituents m is an odd integer.

We solve this system by truncation in Fock space, to $m \leq N_{\max}$, and by expansion of the Fock-state wave functions in terms of multivariate symmetric polynomials $P_{ni}^{(m)}(y_1, \dots, y_m)$ [11, 12]

$$\psi_m(y_1, \dots, y_m) = \sqrt{y_1 y_2 \dots y_m} \sum_{ni} c_{ni}^{(m)} P_{ni}^{(m)}(y_1, \dots, y_m). \quad (2.10)$$

This converts the system into a generalized matrix eigenvalue problem

$$\begin{aligned} & \sum_{n'i'} \left[\begin{pmatrix} +1 \\ -1 \\ 2 \end{pmatrix} T_{ni,n'i'}^{(m)} + g V_{ni,n'i'}^{(m,m)} \right] c_{n'i'}^{(m)} + g \sum_{n'i'} V_{ni,n'i'}^{(m,m+2)} c_{n'i'}^{(m+2)} + g \sum_{n'i'} V_{ni,n'i'}^{(m,m-2)} c_{n'i'}^{(m-2)} \\ & + \sqrt{g} \sum_{n'i'} V_{ni,n'i'}^{(m,m+1)} c_{n'i'}^{(m+1)} + \sqrt{g} \sum_{n'i'} V_{ni,n'i'}^{(m,m-1)} c_{n'i'}^{(m-1)} = \frac{M^2}{\mu^2} \sum_{n'i'} B_{ni,n'i'}^{(m)} c_{n'i'}^{(m)}, \end{aligned}$$

with $g = \lambda/(4\pi\mu^2)$. The matrices T , V and B are defined as

$$T_{ni,n'i'}^{(m)} = m \int \left(\prod_j dy_j \right) \delta(1 - \sum_j y_j) \left(\prod_{j=2}^m y_j \right) P_{ni}^{(m)}(y_j) P_{n'i'}^{(m)}(y_j), \quad (2.12)$$

$$V_{ni,n'i'}^{(m,m)} = \frac{1}{4} m(m-1) \int \left(\prod_j dy_j \right) \delta(1 - \sum_j y_j) \times \int dx_1 dx_2 \delta(y_1 + y_2 - x_1 - x_2) \left(\prod_{j=3}^m y_j \right) P_{ni}^{(m)}(y_j) P_{n'i'}^{(m)}(x_1, x_2, y_3, \dots, y_m), \quad (2.13)$$

$$V_{ni,n'i'}^{(m,m+2)} = \frac{1}{6} m \sqrt{(m+2)(m+1)} \int \left(\prod_j dy_j \right) \delta(1 - \sum_j y_j) \times \int dx_1 dx_2 dx_3 \delta(y_1 - x_1 - x_2 - x_3) \left(\prod_{j=2}^m y_j \right) P_{ni}^{(m)}(y_j) P_{n'i'}^{(m+2)}(x_1, x_2, x_3, y_2, \dots, y_m), \quad (2.14)$$

$$V_{ni,n'i'}^{(m,m+1)} = \sqrt{\frac{3}{2}} m \sqrt{m+1} \int \left(\prod_j dy_j \right) \delta(1 - \sum_j y_j) \times \int dx_1 dx_2 \delta(y_1 - x_1 - x_2) \left(\prod_{j=2}^m y_j \right) P_{ni}^{(m)}(y_j) P_{n'i'}^{(m+1)}(x_1, x_2, y_2, \dots, y_m), \quad (2.15)$$

and

$$B_{ni,n'i'}^{(m)} = \int \left(\prod_j dy_j \right) \delta(1 - \sum_j y_j) \left(\prod_j y_j \right) P_{ni}^{(m)}(y_j) P_{n'i'}^{(m)}(y_j), \quad (2.16)$$

with $V_{ni,n'i'}^{(m,m-2)}$ and $V_{ni,n'i'}^{(m,m-1)}$ obtained as the adjoints of $V_{ni,n'i'}^{(m-2,m)}$ and $V_{ni,n'i'}^{(m-1,m)}$, respectively.

We convert the matrix problem to an ordinary eigenvalue problem by a singular value decomposition (SVD) [18] of the overlap matrix $B^{(m)}$. This is an implicit orthogonalization of the basis. The SVD is $B^{(m)} = U^{(m)} D^{(m)} U^{(m)T}$, where the columns of $U^{(m)}$ are the eigenvectors of $B^{(m)}$ and $D^{(m)}$ is a diagonal matrix of the eigenvalues. We keep in $U^{(m)}$ only those columns associated with eigenvalues of $B^{(m)}$ that are above some positive threshold, in order to eliminate from the basis those combinations of polynomials that are nearly linearly dependent [19]. We then define new vectors of coefficients $\vec{c}^{(m)'} = D^{1/2} U^T \vec{c}^{(m)}$ and new matrices, such as $T^{(m)'} = D^{-1/2} U^T T^{(m)} U D^{-1/2}$. In terms of these, the matrix eigenvalue problem is an ordinary one, which we then diagonalize by standard means.

III. RESULTS

Figure 1 shows the results for the mass squared of the lowest eigenstate as a function of the dimensionless coupling g and for both the symmetric and broken phases. In the symmetric phase, we find a critical coupling of [15] $g = 2.1 \pm 0.05$. In the broken phase, the critical coupling values extrapolate to $g = 0.2 \pm 0.02$.

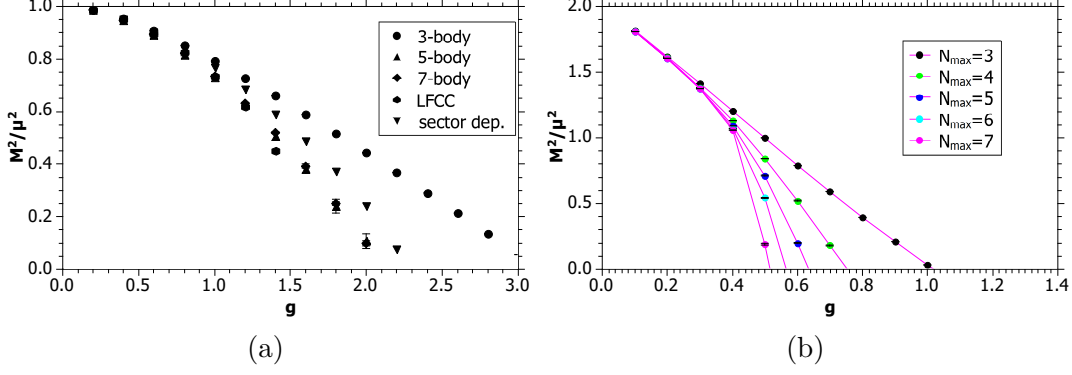


FIG. 1. Mass squared vs coupling strength for the (a) symmetric phase and (b) broken phase. The different Fock-space truncations in (a) are the three-body (triangles), five-body (squares), and seven-body (diamonds) Fock sectors. Results for the light-front coupled-cluster method [20] (circles) are also included. In (b), each set of points corresponds to a different Fock-space truncation to N_{\max} constituents. the different truncations are the four-body (triangles), six-body (squares), and eight-body (diamonds) Fock sectors. Error bars are determined by the fits to extrapolation in the polynomial basis size.

These critical coupling values can be compared to Chang's duality [6, 17, 21, 22], which is obtained from the connection between normal orderings with respect to different masses [23]:

$$N_+[\phi^2] = N_-[\phi^2] + \frac{1}{4\pi} \ln \frac{\mu_+^2}{\mu_-^2}, \quad (3.1)$$

$$N_+[\phi^4] = N_-[\phi^4] + 6 \frac{1}{4\pi} \ln \frac{\mu_+^2}{\mu_-^2} N_-[\phi^2] + 3 \left(\frac{1}{4\pi} \ln \frac{\mu_+^2}{\mu_-^2} \right)^2. \quad (3.2)$$

The Hamiltonian density is then written as

$$\mathcal{H}^- = \left(\frac{1}{2} \mu_+^2 + \frac{\lambda}{4} \frac{1}{4\pi} \ln \frac{\mu_+^2}{\mu_-^2} \right) N_-[\phi^2] + \frac{\lambda}{4!} N_-[\phi^4] + \frac{1}{4\pi} \ln \frac{\mu_+^2}{\mu_-^2} \left(2\mu_+^2 + \frac{\lambda}{8} \frac{1}{4\pi} \ln \frac{\mu_+^2}{\mu_-^2} \right). \quad (3.3)$$

This is equivalent to \mathcal{H}^- with negative mass squared if $\frac{1}{2} \mu_+^2 + \frac{\lambda}{4} \frac{1}{4\pi} \ln \frac{\mu_+^2}{\mu_-^2} = -\frac{1}{2} \mu_-^2$. For the dimensionless couplings $g_{\pm} \equiv \lambda/4\pi \mu_{\pm}^2$ this becomes $\frac{1}{g_+} - \frac{1}{2} \ln g_+ = -\frac{1}{2} \ln(g_-) - \frac{1}{g_-}$. The comparison is illustrated in Fig. 2

The relative probabilities of the higher Fock sectors are readily computed from the Fock-state wave functions obtained in the eigenvector of the Hamiltonian matrix. A plot can be found in [16]. There is no indication of critical behavior at $g = 2.1$, where one would expect that the higher Fock sectors would dominate. A natural assumption for why this might be happening is that the numerical calculation used the same constituent mass μ in every Fock sector, making the invariant mass of the N -constituent Fock sector of order $\sum_i^N \frac{\mu^2}{1/N} = N^2 N \mu^2$. The higher sectors are then suppressed by this large invariant mass.

This can be avoided by the use of a sector-dependent constituent mass [24–29]; our approach is described in [16]. The results for the lowest odd eigenstate are shown in Fig. 3. They are consistent with the sector-independent approach, and the estimate of the critical coupling remains unchanged at a value of 2.1. As can be seen in [16], the relative probabilities also remain nearly the same.

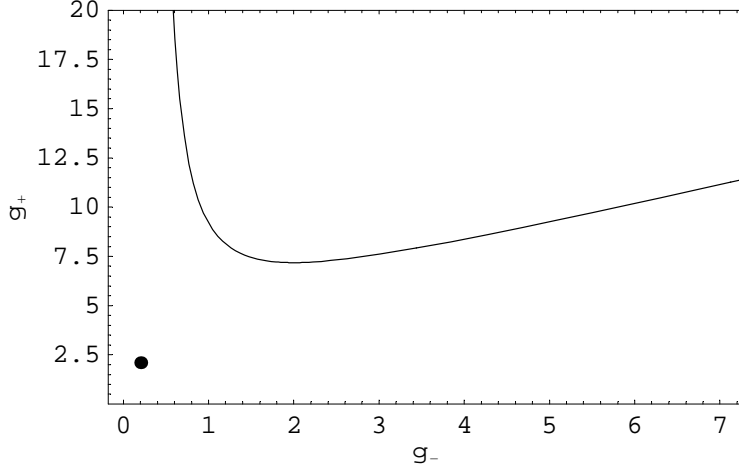


FIG. 2. The duality in couplings between the symmetric and broken phases. The solid line is the semi-classical duality of Chang [22]. The point corresponds to our numerical results.

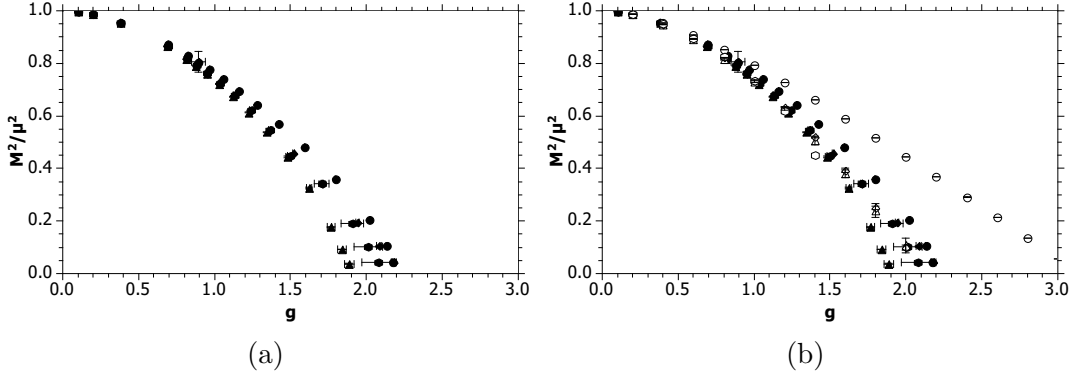


FIG. 3. Mass squared for the lowest eigenstate as a function of the dimensionless coupling g with (a) a sector-dependent constituent mass and (b) with both sector-dependent (filled symbols) and independent (open symbols). For (a), the Fock-space truncations are to a number of constituents $N_{\max} = 3$ (circles), 5 (triangles), 7 (diamonds), and 9 (hex); the error bars estimate the range of fits for the μ_1 extrapolations used to obtain g and M . In (b), the sector-independent, five and seven-constituent results are nearly identical with the nine-constituent sector-dependent results. Plot (b) also includes, as open hexagons, the results from a light-front coupled-cluster calculation [20].

IV. SUMMARY

These calculations have revealed two inconsistencies in the light-front approach to ϕ^4 theory. One is the absence of a vacuum expectation value (VEV) above the critical coupling, where the $\phi \rightarrow -\phi$ symmetry is to be broken, and the presumably related behavior in the explicitly broken phase, where the vacuum expectation value remains nonzero above the critical coupling. A Gaussian effective potential analysis [30, 31] can determine a VEV that does flip between zero and non-zero, but it does so discontinuously, which is inconsistent with the known second-order nature of the transition.

Another inconsistency is the smooth behavior of the computed Fock sector probabilities as the coupling is increased through the critical value, as presented in more detail in [16]. The

relative probabilities for the sector-dependent and independent calculations are essentially the same in the three-body Fock sector. This indicates full convergence with respect to the Fock-space truncation. In Fock sectors with five and seven constituents, the relative probability for the sector-dependent case rises above the probability in the standard case as the critical coupling is approached. This greater probability is expected; however, the full expectation was that these probabilities would increase much more rapidly. The hypothesis, that a sector-dependent mass would reveal the critical behavior, must be incorrect. It seems likely that a coherent-state approach is needed, something that the light-front coupled cluster method can bring [20, 32].

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